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On Hoyle-Narlikar-Wheeler mechanism of vibration energy powered magneto-dipole emission of neutron stars

**Sergey Bastrukov[1,2] • Junwei Yu[1] •
Irina Molodtsova[2] • Renxin Xu[1]**

Abstract We revisit the well-known Hoyle-Narlikar-Wheeler proposition that neutron star emerging in the magnetic-flux-conserving process of core-collapse supernova can convert the stored energy of Alfvén vibrations into power of magneto-dipole radiation. We show that the necessary requirement for the energy conversion is the decay of internal magnetic field. In this case the loss of vibration energy of the star causes its vibration period, equal to period of pulsating emission, to lengthen at a rate proportional to the rate of magnetic field decay. These prediction of the model of vibration powered neutron star are discussed in juxtaposition with data on pulsating emission of magnetars whose radiative activity is generally associated with the decay of ultra strong magnetic field.

Keywords neutron stars, torsion Alfvén vibrations, magneto-dipole radiation

1 Introduction

It is generally known today that the existence of neutron stars with dipole magnetic field of up to $B = 10^{14}-10^{16}$ G, emerging in the magnetic-flux-conserving core-collapse supernova, has first been regarded by Woltjer (1964). Based on this Hoyle, Narlikar and Wheeler (1964) proposed that a neutron star with such a field can generate magneto-dipole radiation by means of conversion of energy of hydromagnetic, Alfvén, vibrations into power of electromagnetic emission (e.g., Pacini 2008). In recent work (Bastrukov et al. 2011) such a possibility has been studied in some details in the context of electromagnetic activity of magnetic white dwarfs. In this article, we present an extensive analysis of vibration-energy powered magneto-dipole radiation of a neutron star undergoing global Alfvén torsion seismic vibrations about axis of its dipole magnetic moment. In §2, the general theory of node-free torsional Alfvén vibrations of perfectly conducting degenerate stellar matter with frozen-in constant-in-time magnetic field is briefly outlined. In §3, the theory is extended to the case of Lorentz-force-driven torsion vibrations in time-varying magnetic field. It is shown that magnetic field decay is crucial (necessary condition) to the energy conversion from Alfvén vibrations to magneto-dipole radiation. The predictions of the model of vibration powered neutron star are discussed in juxtaposition with available data . The results are summarized in §4 with emphasis on the relevance of the model to electromagnetic activity of magnetars.

Sergey Bastrukov
Junwei Yu
Irina Molodtsova
Renxin Xu
State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
Joint Institute for Nuclear Research, 141980 Dubna, Russia
[1]State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
[2]Joint Institute for Nuclear Research, 141980 Dubna, Russia

2 Alfvén node-free vibrations in constant-in-time frozen-in magnetic field

The starting point in the study of node-free Alfvén (non-compression magneto-mechanical) vibrations of a neutron star (thought of as a spherical solid mass of a perfectly conducting non-flowing continuous matter with frozen-in constant-in-time magnetic field) are equations of magneto-solid-mechanics (Bastrukov et al.

2009a, 2009b, Molodtsova et al. 2010)

$$\rho\ddot{\mathbf{u}} = \frac{1}{c}[\delta\mathbf{j}\times\mathbf{B}], \tag{1}$$

$$\delta\mathbf{j} = \frac{c}{4\pi}[\nabla\times\delta\mathbf{B}], \quad \delta\mathbf{B} = \nabla\times[\mathbf{u}\times\mathbf{B}], \tag{2}$$

$$\rho\,\ddot{\mathbf{u}} = \frac{1}{4\pi}[\nabla\times[\nabla\times[\mathbf{u}\times\mathbf{B}]]]\times\mathbf{B}, \quad \nabla\cdot\mathbf{u} = 0. \tag{3}$$

In such vibrations the rate of differentially rotational material displacements of stellar matter is described by nodeless toroidal vector field

$$\dot{\mathbf{u}}(\mathbf{r},t) = [\boldsymbol{\omega}(\mathbf{r},t)\times\mathbf{r}], \quad \boldsymbol{\omega}(\mathbf{r},t) = [\nabla\chi(\mathbf{r})]\,\dot{\alpha}(t), \tag{4}$$

$$\nabla^2\chi(\mathbf{r}) = 0, \quad \chi(\mathbf{r}) = f_\ell(r)\,P_\ell(\cos\theta), \tag{5}$$

$$f_\ell(r) = A_\ell\,r^\ell + B_\ell\,r^{-\ell-1} \tag{6}$$

which is identical to that for torsion node-free vibrations restored by Hooke's elastic force (Bastrukov et al. 2007a, 2007b). In the last equation, $P_\ell(\cos\theta)$ stands for Legendre polynomial of degree ℓ specifying the overtone of toroidal a-mode; the amplitude $\alpha(t)$ describes time evolution of vibrations (both global and locked in the crust). In computing discrete spectrum of such vibrations, the magnetic field can be conveniently represented in the form: $\mathbf{B}(\mathbf{r}) = B\,\mathbf{b}(\mathbf{r})$, where B is the intensity and $\mathbf{b}(\mathbf{r})$ is dimensionless vector-function of magnetic field distribution over the star volume. Similar representation can be used for the bulk density $\rho(r) = \rho\phi(r)$, where ρ is the density at the star center and $\phi(r)$ describes the radial profile of density which can be taken from computations of neutron star structure relying on realistic equations of state accounting for non-uniform mass distribution in the star interior (e.g., Weber 1999).

Scalar product of (3) with the following separable representation of $\mathbf{u}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\,\alpha(t)$ and integration over the star volume leads to equation for $\alpha(t)$ having the form of equation of harmonic oscillator

$$\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}_m\alpha(t) = 0,\ \mathcal{M} = \rho\,m_\ell,\ \mathcal{K}_m = \frac{B^2}{4\pi}\,k_\ell, \tag{7}$$

$$m_\ell = \int\phi(r)\,\mathbf{a}(\mathbf{r})\cdot\mathbf{a}(\mathbf{r})\,d\mathcal{V},$$

$$k_\ell = \int\mathbf{a}(\mathbf{r})\cdot[\mathbf{b}(\mathbf{r})\times[\nabla\times[\nabla\times[\mathbf{a}(\mathbf{r})\times\mathbf{b}(\mathbf{r})]]]]\,d\mathcal{V},$$

$$\omega_\ell = \sqrt{\frac{\mathcal{K}_m}{\mathcal{M}}} = \omega_A\,\eta_\ell,\ \omega_A = \frac{v_A}{R} = B\sqrt{\frac{R}{3M}},$$

$$\eta_\ell = \sqrt{\frac{k_\ell}{m_\ell}}\,R,\ v_A = \frac{B}{\sqrt{4\pi\rho}}.$$

The amplitude of oscillations with constant in time frequency ω_ℓ is given by

$$\alpha(t) = \alpha_0\cos\omega t, \quad \alpha_0^2 = \frac{2\bar{E}_A}{\mathcal{M}\omega^2} = \frac{2\bar{E}_A}{\mathcal{K}_m}, \tag{8}$$

$$\begin{aligned}\bar{E}_{\rm A} &= (1/2)\mathcal{M}\bar{\dot{\alpha}^2} + (1/2)K\bar{\alpha^2}\\ &= (1/2)\mathcal{M}\omega^2\alpha_0^2 = (1/2)K\alpha_0^2\end{aligned} \tag{9}$$

where over-bar stands for averaging over period of vibrations: $\bar{\alpha^2}(t) = (1/2)\alpha_0^2$. This suggests, if all the energy $E_{\rm burst}$ of x-ray outburst goes in the quake-induced vibrations and, i.e. when $E_{\rm burst} = E_A$, the amplitude α_0 can be extracted from the last equation. In the reminder of the paper we remove index ℓ and confine our analysis to the case of quadrupole overtone of a-mode, i.e., putting $\omega = \omega_{\ell=2}$. The important outcome of assumption about constant in time undisturbed magnetic field is the vibration energy conservation

$$\frac{dE_A(t)}{dt} = 0, \quad E_A(t) = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}_m\alpha^2(t)}{2} \tag{10}$$

and also that the fundamental frequency $\nu_A = \omega_A/2\pi$ (where $\omega_A = v_A/R$) and the period $P_A = \nu_A^{-1}$ of global Alfvén oscillations

$$\nu_A = \frac{B}{2\pi}\sqrt{\frac{R}{3M}}, \quad P_A = \frac{2\pi}{B}\sqrt{\frac{3M}{R}} \tag{11}$$

remain constant in time. The magnitudes of basic frequency of toroidal a mode in neutron stars with magnetic fields typical to radio-pulsars $B_{12} = B/(10^{12}$ G and magnetars (soft gamma repeaters) $B_{14} = B/(10^{14}$ G are

$$\nu_{\rm A}(B_{12}) = 2.055\cdot10^{-3}B_{12}R_6^{1/2}(M/M_\odot)^{-1/2}, \tag{12}$$

$$\nu_{\rm A}(B_{14}) = 0.2055B_{14}R_6^{1/2}(M/M_\odot)^{-1/2} \tag{13}$$

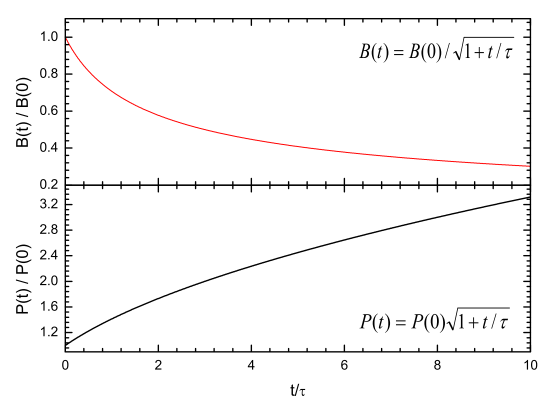

Fig. 1 The magnetic field decay and period elongation in a quaking neutron star undergoing Lorentz-force-driven quadrupole torsion vibrations caused by conversion of vibration energy into energy of magneto-dipole emission.

where $R_6 = R/(10^6$ cm. It should be noted that above equations of solid-star model are applicable not only to neutron stars but also for white dwarfs (Molodtsova et al. 2010; Bastrukov et al. 2010) and quark stars (Xu 2003, 2009). In this work, continuing investigations reported in (Bastrukov et al. 2009a, 2009b, 2010), we relax the assumption about constant-in-time undisturbed magnetic field and examine the impact of its decay on quake-induced vibrations and resultant magneto-dipole radiation.

3 Energy conversion from Alfvén vibrations into magneto-dipole radiation

In the model under consideration, the decay of magnetic field (during the time of post-quake vibrational relaxation of the star) is though of as caused by coupling of the vibrating star with outgoing material expelled by quake. With allow for dependence on time of undisturbed magnetic field intensity, $B = B(t)$, the above chain of argument leads to equation of vibrations with the spring constant depending on time: $\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}_m(B(t))\alpha(t) = 0$. The main subject of our further analysis is the impact of magnetic field decay on radiative process of neutron star undergoing Lorentz-force-driven vibrations obeying the last equation. It is easy to see that in this case the energy of vibrations is not conserved. The loss of vibration energy $E_A = (1/2)[\mathcal{M}\dot{\alpha}^2(t) + \mathcal{K}_m(B(t))\alpha^2(t)$ is determined by the rate of magnetic field decay

$$\frac{dE_A(t)}{dt} = \frac{\alpha^2(t)}{2}\frac{d\mathcal{K}_m(B)}{dB}\frac{dB(t)}{dt}. \quad (14)$$

In the reminder we focus on the conversion of energy of Alfvén vibrations into power of its magneto-dipole radiation which is described by equation

$$\frac{dE_A(t)}{dt} = -\frac{2}{3c^3}\delta\ddot{\mathbf{m}}^2. \quad (15)$$

Let us consider torsional magneto-mechanical oscillations which are accompanied by fluctuations of total magnetic moment $\delta\mathbf{m}(t)$ preserving its initial (in seismically quiescent state) direction $\mathbf{m} = m\,\mathbf{n} = \text{constant}$, i.e. a case when $\delta\mathbf{m}(t)||\mathbf{m}$. The total magnetic dipole moment should execute oscillations with frequency $\omega(t)$ equal to that for magneto-mechanical vibrations of stellar matter which are described by equation for $\alpha(t)$. This means that $\delta\mathbf{m}(t)$ and $\alpha(t)$ must be subjected to equations with similar form, namely

$$\delta\ddot{\mathbf{m}}(t) + \omega^2(t)\delta\mathbf{m}(t) = 0, \quad (16)$$

$$\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = B^2(t)\frac{R\eta^2}{3M}. \quad (17)$$

It is easy to see that equations (16) and (17) can be reconciled if

$$\delta\mathbf{m}(t) = \mathbf{m}\,\alpha(t). \quad (18)$$

With account of all this, the equation (15) is reduced to the equation of the magnetic field decay

$$\frac{dB(t)}{dt} = -\gamma\,B^3(t) \quad \to \quad B(t) = \frac{B(0)}{\sqrt{1+t/\tau}}, \quad (19)$$

$$\tau = [2\gamma B^2(0)]^{-1}, \quad \gamma = m^2\frac{2R\eta^2}{9M\mathcal{M}c^3}. \quad (20)$$

This shows that duration of decay, τ strongly depends upon intensity of initial field $B(0)$: the larger $B(0)$, the shorter decay time τ.

3.1 Magnetic-field-decay-induced lengthening of pulse period

The most striking effect of magnetic field decay on vibration-powered radiation is the lengthening of periods of pulsating magneto- dipole radiation, as is demonstrated in Fig.1. The period and its derivative are given by

$$P(t) = P(0)\,\sqrt{1+(t/\tau)}, \; \dot{P}(t) = \frac{P(0)}{2\tau\sqrt{1+(t/\tau)}},$$

$$\tau = \frac{P^2(0)}{2P(t)\dot{P}(t)}, \quad P^2(0) = \frac{4\pi^2}{B^2(0)}\frac{3M}{R\eta^2}. \quad (21)$$

Equating two independent estimates of τ, given by equations (20) and (21), one finds that relation between magnitude of total magnetic moment of the star m and $P(t)$ and $\dot{P}(t)$ is given by

$$m = A\sqrt{P(t)\dot{P}(t)}, \quad A = \sqrt{\frac{3\mathcal{M}c^3}{8\pi^2}} = \text{constant}. \quad (22)$$

For a neutron star undergoing quadrupole node-free torsion Alfvén vibrations about axis of dipole magnetic moment, the mass parameter $\mathcal{M}$ (represented in terms of moment of inertia $I = (2/5)MR^2$), the magneto-mechanical stiffness $\mathcal{K}$, frequency ν are given by

$$\mathcal{M} = \frac{9}{7}I = \frac{18}{35}MR^2, \; \mathcal{K} = \frac{6}{5}B^2\,R^3, \; \omega = \sqrt{\frac{\mathcal{K}}{\mathcal{M}}} \quad (23)$$

$$\nu(t) = \frac{\omega(t)}{2\pi} = \frac{B(t)}{2\pi}\sqrt{\frac{7R}{3M}}. \quad (24)$$

In these equations, the star mass M and radius R serve as input parameters. For a neutron star with $M = 1.4\,M_\odot$ and radius $R = 10$ km, we obtain

$$\nu(t) = 4.6\times10^{-15}B\;(M/M_\odot)^{-1/2}R_6^{1/2}, \;\; \text{Hz} \quad (25)$$

$$m = 3.8\times10^{37}\,\sqrt{P(t)\,\dot{P}(t)}, \; \text{G cm}^3. \quad (26)$$

These are unique to a neutron star emitting pulsed magneto-dipole radiation at the expense of energy of torsion Alfvén vibrations. Taking from observations $P(t) = \nu^{-1}(t)$ and $\dot{P}(t)$ one can extract intensity of magnetic field and the magnitude of total dipole magnetic moment of the star. Knowing B and m, from equation 20 one can obtain the time of magnetic field decay τ. The practical usefulness of these estimates is that they can be used as a guide in search for fingerprints of vibration powered neutron stars.

The basic conclusion of the model regarding the lengthening of vibration period (equal to period of electromagnetic pulse) can be demonstrated by solution of equation for vibration amplitude

$$\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = \frac{\omega^2(0)}{1 + t/\tau}. \tag{27}$$

Making us of new variable, $s = 1 + t/\tau$, the above equation takes the form

$$s\alpha''(s) + \beta^2\alpha(s) = 0, \quad \beta^2 = \omega^2(0)\tau^2 \tag{28}$$

whose analytic solution reads (e.g., Polyanin and Zaitsev, 2004)

$$\alpha(s) = s^{1/2}\{C_1\, J_1(2\beta s^{1/2}) + C_2 Y_1(2\beta s^{1/2})\} \tag{29}$$

where $J_1(2\beta s^{1/2})$ and $Y_1(2\beta s^{1/2})$ are Bessel functions (Abramowitz and Stegun 1972). The arbitrary constants C_1 and C_2 can be eliminated from the following boundary conditions $\alpha(t = 0) = \alpha_0$, $\alpha(t = \tau) = 0$ where α_0 the above defined amplitude at the initial state of vibrations (before magnetic field decay). As a result, the solution of (28) can be represented in the form

$$\alpha(t) = C\,[1 + (t/\tau)]^{1/2} \tag{30}$$
$$\times\{J_1(2\beta\,[1 + (t/\tau)]^{1/2}) - \eta\, Y_1(2\beta\,[1 + (t/\tau)]^{1/2})\},$$
$$\eta = \frac{J_1(z(\tau))}{Y_1(z(\tau))}, \quad C = \alpha_0[J_1(z(0)) - \eta\, Y_1(z(0))]^{-1}.$$

The lengthening of vibration period is illustrated in Fig. 2. in which the vibration amplitude is plotted as a function of $x = t/\tau$ with fixed value of C and different values of parameters η and β, respectively. This figure shows that η is the parameter regulating magnitude of amplitude: the larger η the higher the amplitude. However, this parameter does not affect the rate of period lengthening. Both, the elongation rate of vibration period and magnitude of vibration amplitude are highly sensitive to parameter β.

It worth emphasizing that the magnetic-field-decay-induced loss of vibration energy is substantially different from the vibration energy dissipation caused by shear viscosity of matter resulting in heating of stellar material. The characteristic feature of this latter mechanism of vibration energy conversion into the heat (i.e., into the energy of non-coherent electromagnetic emission responsible for the formation of photosphere of the star) is that the frequency and period of vibrations are the same as that in the case of viscous-free vibrations (Bastrukov et al. 2010). However, it is no longer so in the case under consideration. It follows from above that the magnetic field decay resulting in the loss of total energy of Alfvén vibrations of the star causes its vibration period to lengthen at a rate proportional to the rate of magnetic field decay.

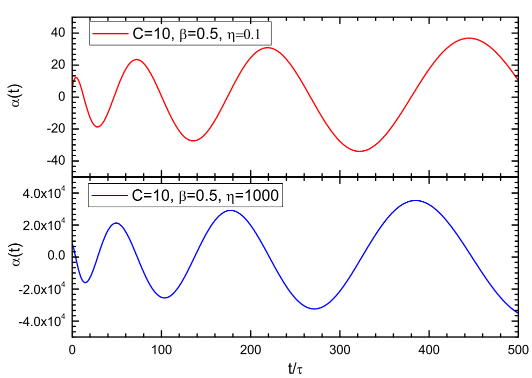

Fig. 2 Time evolution of the vibration amplitude, $\alpha(t)$ computed with indicated with indicated parameters C, β and η.

3.2 Comparison with model of rotation-energy powered neutron star

All the above shows that in the model of vibration-energy powered magneto-dipole emission under consideration, the equation of magnetic field evolution is obtained in similar fashion as equation for the angular velocity $\Omega(t)$ does in the standard model of rotation-energy powered magneto-dipole radiation

$$\frac{dE}{dt} = -\frac{2}{3c^3}\delta\ddot{\mathbf{m}}^2(t), \tag{31}$$

$$E = E_R(t) = \frac{1}{2}I\,\Omega^2(t), \quad I = \frac{2}{5}M\,R^2 \tag{32}$$

One of the basic postulates of the model of rotation-energy powered emission is that the time evolution of $\delta\mathbf{m}$ is governed by the equation

$$\delta\ddot{\mathbf{m}}(t) = [\boldsymbol{\Omega}(t) \times [\boldsymbol{\Omega}(t) \times \mathbf{m}]], \quad \mathbf{m} = \text{constant}, \tag{33}$$

$$\delta\ddot{m}^2 = m_\perp^2\Omega^4, \quad m_\perp = m\sin\theta \tag{34}$$

where θ is angle of inclination of $\mathbf{m}$ to $\mathbf{\Omega}(t)$. The utilized in this model interrelation between the undisturbed total magnetic moment $\mathbf{m}$ and constant magnetic field $\mathbf{B}$ frozen in the star presumes that the neutron star matter is in the state of permanent magnetization $\mathbf{M}$ of non-ferromagnetic type: $\mathbf{M} = (3/8\pi)\,\mathbf{B}$ (see, for instance, Landau, Lifshitz, Pitaevskii 1995 [§76, problem 2]), namely

$$m = \int M\, d\mathcal{V} = \frac{1}{2} B\, R^3 \quad \rightarrow \quad B = \frac{2m}{R^3}. \tag{35}$$

This estimate shows that in rotation powered neutron star, the frozen-in the star magnetic field operates like a passive promoter of magneto-dipole radiation, that is, remain constant in the process of radiation. As a result, the equation of energy conversion from rotation to radiation is reduced to equation for rotation frequency

$$\dot{\Omega}(t) = -K\Omega^3(t), \quad K = \frac{2m_{\perp}^2}{3Ic^3}, \tag{36}$$

$$\Omega(t) = \frac{\Omega(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\,K\,\Omega^2(0). \tag{37}$$

Thus, in the model of vibration-energy powered emission the elongation of pulse period (and, hence, the origin of $\dot{P}$) is attributed to magnetic field decay, whereas in the model of rotation-energy powered emission the lengthening of pulse period is ascribed to the slow down of the neutron star rotation (e.g., Manchester & Taylor 1977, Lorimer & Kramer 2004, Bisnovatyi-Kogan 2010).

4 Summary

Since celebrated hypothesis of Baade-Zwicky about birth of neutron stars in core- collapse supernova and Woltier-Ginzburg proposition that they should emerge as sources of super strong dipole magnetic fields, it has been realized that such a field should operate as a chief promoter of magneto-dipole electromagnetic emission whose energy supply can be maintained either by energy of rotation or by energy of Alfvén vibrations (e.g., Pacini 2008). As we mentioned, the idea that neutron stars can radiate like Hertzian magnetic dipole at the expense of energy of hydromagnetic vibrations has first been considered by Hoyle et al. (1964), but without discussion of peculiarities of vibration-energy powered emission. With this in mind, one of the prime goals of this work was to explore some characteristic features of such emission. What is newly disclosed here is that necessary condition for the energy conversion from Alfvén torsion vibrations into magneto-dipole radiation is the decay of internal magnetic field. The basic prediction of developed theory is the lengthening of periods of pulsating vibration-energy powered magneto-dipole radiation which is caused by decay of internal magnetic field. Taking into account that both models of rotation powered and vibration powered pulsating emission of neutron stars predict lengthening of pulse period it is of interest to single out the potential subclass of objects whose pulsating radiation could be explained by vibrations, rather than rotation. In so doing, in Table 1 we present estimates of ν_A and τ as functions of increasing intensity of magnetic field for neutron stars with typical masses and radii. It is seen that both above characteristics are quite sensitive to M and R and, thus, to equation of state of stellar matter. For magnetic fields of typical radio pulsars, $B \sim 10^{12}$ G, the computed frequency ν_A is much smaller than the detected frequency of pulses. This shows that considered mechanism of vibration powered pulsating radiation has nothing to do with pulsating emission of radio- pulsars. In the meantime, for neutron stars endowed with magnetic fields $B \sim 10^{14}$ G our estimates of ν_A fall in the realm of observed frequencies of high-energy pulsating emission of soft gamma repeaters (SGRs), anomalous X-ray pulsars (AXPs) and sources exhibiting similar features. According to common belief, these are magnetars - highly magnetized neutron stars whose radiative activity is related with magnetic field decay (e.g., Woods and Thompson 2006). As was emphasized, the decay time of magnetic field (and, hence, the duration of vibration powered pulsating radiation) strongly depend on the intensity of initial magnetic field of the star: the larger magnetic field the shorter its decay time. Also, the magnitude of τ indicate that definite conclusion about secular lengthening of pulse period of vibration powered neutron star can be made only on the basis of reliable statistics which demands fairly long monitoring time, namely, several years if not decades. All these features of the vibration powered radiation of a neutron star are consistent with data on long-term monitoring activity of AXP, 1E 2259.1+586 (2 decades, 1980-2000) with using Rossi X-ray Timing Explorer (RXTE) (Gavriil et al. 2004). Similar trend in

Table 1 Basic frequency of Alfvén vibrations, ν_A, and the time of magnetic field decay, τ, computed with parameters of typical pulars and magnetars.

$M(M_\odot)$	R(km)	B(G)	ν_A(Hz)	τ(yr)
0.8	20	10^{12}	$3.25\,10^{-3}$	$4.53\,10^{10}$
1.0	15	10^{13}	$2.52\,10^{-2}$	$2.98\,10^{7}$
1.1	13	10^{14}	0.22	$7.4\,10^{3}$
1.3	11	10^{15}	1.89	2.38

decreasing frequency of long-periodic pulsating emission has been revealed in RXTE observations of the source XTE J1810-197 (Ibrahim et al. 2004). However, the opposite tendency (increase of pulse frequency and, hence, shortening of period) has been disclosed in activity of the X-ray source CXO J164710.2-455216 (Muno et al. 2006) after short time of its monitoring (May and July, 2005) with using of Chandra facility. The most plausible reason of this discrepancy (with both, the above observations and theoretical expectations of considered here models) is too short time of monitoring or, in other words, the lack of reliable statistics. With all above, we conclude that vibration powered neutron stars, if exist, can be revealed among the magnetars – AXPs/SGRs - like sources whose persistent X-ray luminosity, $10^{34} < L_X < 10^{36}$ ergs s^{-1}, cannot be explained by the loss of energy of slow rotation.

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